Coupled pendula with Cobra SMARTsense advanced version

Physics | Acoustics | Wave Motion

Difficulty level	Group size	Preparation time	Execution time
-	-	-	-

This content can also be found online at:

http://localhost:1337/c/608eb52776e9660003ec347e

General information

Application

Setup

Pendulum oscillations offer a first understanding of mechanical systems close to the harmonic oscillator, which is fundamental in the description of many physical systems in fields such as particle physics and solid state physics.

This experiment investigates the behaviour of such a system using coupled pendula.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Other information (1/2)

PHYWE

Prior knowledge

The behaviour of a singular pendulum should be known.

Scientific principle

Two identical gravity pendula with a particular characteristic frequency are coupled via a “soft” spiral spring. The amplitudes of both pendula are recorded as a function of time. The coupling factors are determined by way of different methods. Then, the local points of the oscillation are integrated into the video.

Other information (2/2)

PHYWE

Learning objective

The goal of this experiment is to investigate the oscillation behaviour of coupled pendula.

Tasks

1. Determination of the spring constant of the coupling spring.
2. Determination and adjustment of the characteristic frequency of the uncoupled pendulum. Determination of the moment of inertia of the pendulum.
3. Graphical representation of the oscillation of the two pendula as a function of time and determination of the oscillation frequency compared to the theoretical oscillation frequency for A) the “in phase” oscillation. B) the “antiphase” oscillation. C) the beat case.

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Theory (1/8)

PHYWE

If two gravity pendula P_1 and P_2 with the same characteristic frequency ω_0 are coupled by way of a spring, the following is true for the torque in the case of the position of rest and in the case of small deflections due to gravity and due to the spring tension (see Figure 1):

Torque due to gravity:

$$M_{s,0} = mgL\sin(\Phi_0) \approx mgL\Phi_0 \qquad (1)$$

Torque due to spring tension:

$$M_{F,0} = -D_F x_0 l \cos(\Phi_0) \approx -D_F x_0 l$$

D_F Spring constant, x_o Elongation of the spring, l coubling strength, m Mass of the pendulum, L Length of the pendulum, g gravitational acceleration, Φ_0 Angle between the vertical and the position of rest

Theory (2/8)

PHYWE

If P_1 is deflected by D_F and P_2 by Φ_0 and if both pendula are released simultaneously, the following results because of

$$I\ddot{\Phi} = M$$

I = Moment of inertia of the pendulum around its pivot

$$I\ddot{\Phi}_1 = M_1 = -mgL\Phi_1 + D_f l^2(\Phi_2 - \Phi_1) \qquad (2)$$

$$I\ddot{\Phi}_2 = M_2 = -mgL\Phi_2 + D_f l^2(\Phi_2 - \Phi_1)$$

Following the introduction of the abbreviations

$$\omega_0^2 = \frac{mgl}{I} \text{ and } \Omega^2 = \frac{D_F l^2}{I} \qquad (3)$$

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Theory (3/8)

PHYWE

We obtain the following based on equation (2)

$\ddot{\Phi}_1 + \omega_0^2 \Phi_1 = -\Omega^2(\Phi_2 - \Phi 1))$ (4)

$\ddot{\Phi}_2 + \omega_0^2 \Phi_1 = +\Omega^2(\Phi_2 - \Phi 1))$

With, t=0 the following three initial conditions are successfully realised.

A) "in phase" oscillation

$\Phi_1 = \Phi_2 = \Phi_A; \Phi_1 - \Phi_2 = 0$

B) “antiphase” oscillation

$-\Phi_1 = \Phi_2 = \Phi_A; \Phi_1 - \Phi_2 = 2\Phi$

C) Beat case

$\Phi_1 = \Phi_A; \Phi_2 = 0; \Phi_1 - \Phi_2 = \Phi_A$

The general solutions of the differential equations (4) with the initial conditions (5) are:

A: $\Phi_1(t) = \Phi_2(t) = \Phi_A \cos(\omega_0 t)$ (6a)

B: $\Phi_1(t) = \Phi_A \cos(\sqrt{\omega_0^2 + 2\Omega^2 t})$ (6b)

$\Phi_2(t) = -\Phi_A \cos(\sqrt{\omega_0^2 + 2\Omega^2 t})$

Theory (4/8)

PHYWE

$$\Phi_1(t) = \Phi_A \cos\left(\frac{\sqrt{\omega_0^2+2\Omega^2}-\omega_0}{2} \cdot t\right) \cdot \cos\left(\frac{\sqrt{\omega_0^2+2\Omega^2}+\omega_0}{2} \cdot t\right) \quad (6c)$$

$$\Phi_2(t) = \Phi_A \cos\left(\frac{\sqrt{\omega_0^2+2\Omega^2}-\omega_0}{2} \cdot t\right) \cdot \sin\left(\frac{\sqrt{\omega_0^2+2\Omega^2}+\omega_0}{2} \cdot t\right)$$

Note

A. “in phase” oscillation.

Both pendula oscillate in phase and with the same amplitude and frequency ω_G. The latter is identical with the characteristic frequency of the ω_0 uncoupled pendulum.

$\omega_g = \omega_0$ (7a)

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Theory (5/8)

PHYWE

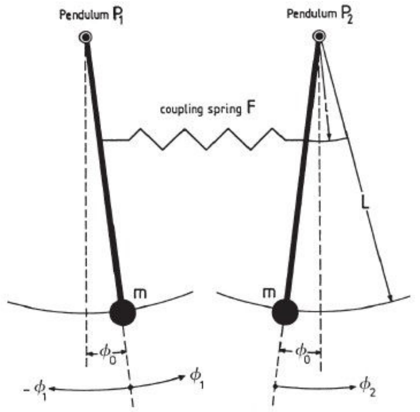

Figure 1: Diagram indicating the various names that are used in the context of coupled pendula.

B. “antiphase” oscillation.

Both pendula oscillate with the same amplitude and frequency ω_C, but with a phase difference of π. In accordance with (3), the angular frequency

$$\omega = \sqrt{\omega_0^2 + 2\Omega^2} \quad \text{(7c)}$$

depends on the length of the pendulum l.

Theory (6/8)

PHYWE

C. Beat case

For a weak coupling, e.g., $\omega_0 \gg \Omega$ the angular frequency of the first factor can be expressed as follows:

$$\omega_1 = \frac{\sqrt{\omega_0^2+2\Omega^2}-\omega_0}{2} \approx \frac{\Omega^2}{2\omega_0} \quad \text{(8a)}$$

For the angular frequency of the second factor, we get:

$$\omega_1 = \frac{\sqrt{\omega_0^2+2\Omega^2}+\omega_0}{2} \approx \omega_0 \frac{\Omega^2}{2\omega_0} \quad \text{(8b)}$$

As a result, we obtain the following:

$$\omega_1 < \omega_2$$

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Theory (7/8)

Figure 1 shows the amplitudes $\Phi_1(t)$ and $\Phi_2(t)$ of both pendula as a function of time for the beat case and for different coupling lengths. As the coupling factor, we define the ratio

$K = \frac{D_F l^2}{mgL + D_F l^2}$ (9)

From equations (3) and (9) we get

$K = \frac{\Omega^2}{\omega_0^2 + \Omega^2}$ (10)

The coupling factor K from equation (10) can be calculated based on the frequencies of the individual oscillation modes. Substituting equations (7a) and (7b) in equation (10) results in

$K = \frac{\omega_c^2 - \omega_g^2}{\omega_c^2 + \omega_g^2}$ ("antiphase" oscillation) (11)

Theory (8/8)

Substituting equations (8a) and (8b) in equation (10) results in

$K = \frac{2\omega_1 \omega_2}{\omega_1^2 + \omega_2}$ (12)

In order to test the influence of the coupling length on the frequencies of the individual oscillations, we substitute equations (11) and (12) in equation (9). This yields the following for the "antiphase" oscillation:

$\omega_1^2 = \frac{2 D_F \omega_0^2}{mgL} l^2 + \omega_0^2$ (13)

The following results for the beat case:

$\omega_1 = \omega_0 \frac{D_F}{2mgL} l^2$ (14)

and $\omega_2 = \omega_0 \frac{D_F}{2mgL} l^2 + \omega_0$ (15)

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Equipment

Position	Material	Item No.	Quantity
1	Pendulum with recorder connection	02816-00	2
2	Helical spring, 3 N/m	02220-00	1
3	Rod with hook	02051-00	1
4	Weight holder, 10 g	02204-00	1
5	Slotted weight, black, 10 g	02205-01	5
6	Cobra SMARTsense - Voltage, ± 30 V (Bluetooth + USB)	12901-01	2
7	USB charger for Cobra SMARTsense and Cobra4	07938-99	1
8	measureLAB, multi-user license	14580-61	1
9	PHYWE Power supply, 230 V, DC: 0...12 V, 2 A / AC: 6 V, 12 V, 5 A	13506-93	1
10	Bench clamp expert	02011-00	2
11	Support rod, stainless steel, 750 mm	02033-00	2
12	Right angle clamp expert	02054-00	4
13	Measuring tape, l = 2 m	09936-00	1
14	Connecting cord, 32 A, 1000 mm, red	07363-01	4
15	Connecting cord, 32 A, 1000 mm, blue	07363-04	4
16	Support rod, stainless steel, 500 mm	02032-00	1

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

PHYWE

Setup and Procedure

Setup and Procedure (1/6)

PHYWE

The Cobra SMARTsense Photogate and measureAPP are required to perform the experiment. The app can be downloaded for free from the App Store - QR codes see below. Check whether Bluetooth is activated on your device (tablet, smartphone).

measureAPP for Android operating systems

measureAPP for iOS operating systems

measureAPP for Tablets / PCs with Windows 10

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Setup and Procedure (2/6)

PHYWE

Fig. 2: Experiment set-up

Prior to starting the measurements, the exact value of the spring constant D_F of the coupling spring must be determined. For this purpose, the spring is placed on the table, fixed on one side and extended at the other side by means of the dynamometer. In doing so the required force F can be read off the dynamometer and the elongation x is measured with the measuring tape. The spring constant can be easily determined from Hooke's law:

$$D_F = \frac{F}{x}$$

It is recommended to lengthen the spring by 10 cm, 20 cm and 30 cm in order to obtain a mean value for the spring constant D_F.

Setup and Procedure (3/6)

PHYWE

Set up both pendula without the coupling spring as shown in Figure 2.

In order to cause the pendula to oscillate, touch the rods of the pendula in the upper third with the tips of your finger and displace them simultaneously in the same or opposite directions until the desired amplitude is reached. In this way, transverse oscillations can be avoided. In view of the subsequent experiment, it should be ensured that the pendula oscillate in the same plane.

The videos are used to determine the period of oscillation $T_=$ for each pendulum. The value of the period of oscillation T of both pendula must be identical. If there are deviations, the lengths of the pendula must be corrected.

For the experiment with the coupling spring, connect the spring to the two plastic hooks on the pendulum rods in two positions. These positions must be equidistant from the pivot of the pendulum. Measure the amplitudes as a function of time with the following initial conditions:

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Setup and Procedure (4/6)

PHYWE

- Both pendula are deflected in the same direction with the same amplitude and they are released simultaneously (“in phase” oscillation).
- Both pendula are deflected with the same amplitude, but in opposite directions (“antiphase” oscillation), and they are released simultaneously.
- One pendulum remains at rest. The second pendulum is deflected and released (beat case). In this case, satisfactory results can only be achieved if the pendula are properly re-adjusted during the preparation phase so that they actually have the same period of oscillation $\bar{T}_0$.

In all three cases, the oscillations must be recorded for at least one to three minutes. Then, the average values of the corresponding periods of oscillation can be determined based on the plotted curves.

Setup and Procedure (5/6)

PHYWE

In terms of the video that will be recorded, the following must be taken into consideration concerning the setting and positioning of the camera:

- Set the number of frames per second to approximately 30 fps.
- Select a light-coloured, homogeneous background.
- Provide additional lighting for the experiment.
- The experiment set-up should be in the centre of the video. To ensure this, position the video camera on a tripod centrally in front of the experiment set-up.

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Setup and Procedure (6/6)

PHYWE

- The experiment set-up should fill the video image as completely as possible.
- The optical axis of the camera must be parallel to the experiment set-up.
- For scaling, the length of the pendulum arm must be measured.

Then, the video recording process and the experiment can be started.

PHYWE

Evaluation

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Task 1 (1/2)

PHYWE

Open the current version of the 'measureLAB' software.

It is available for download at https://www.phywe.com. If you have already set up the experiment, initiate the Bluetooth connection for both sensors at this point. Alternatively, you can connect both voltage sensors to their respective measuring computers via USB. After opening, select the option 'Start with experiment.' Please enter the experiment number P2132567 'Coupled pendula with Cobra SMARTsense advanced version' here. The experiment will now load, indicated by the red hourglass in the bottom left corner. A graph field and two fields for voltage measurement will appear. When both sensors are connected to the software, this will be signaled by a green checkmark.

Task 1 (2/2)

PHYWE

Figure 3 general fundamentals of spring constant. It shows the linear relationship between the mass and the elongation. The gradient of the line is 0.2862. This means that the

spring constant D_F is: 2.86 N/m.

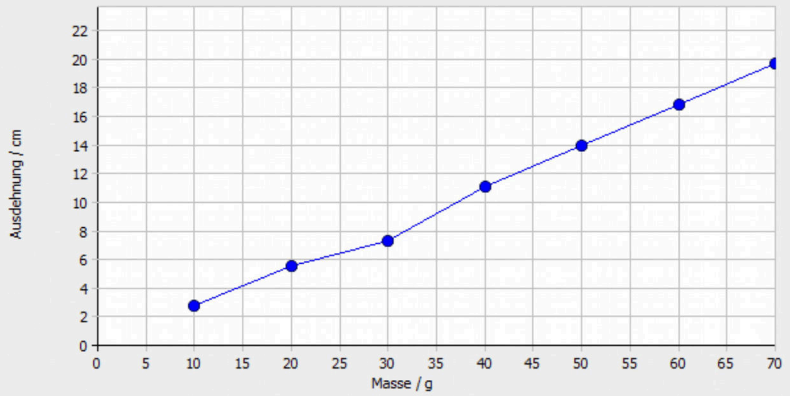

Figure 3: Representation of the elongation of the spring as a function of the mass to which it is subjected.

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Task 2 (1/2)

PHYWE

Move one of the pendulums and then start the measurement immediately by pressing the start button in the bottom left corner.

The experiment begins with the deflection of the pendulum and it ends after several pendulum oscillations. For this experiment, it is important to ensure that the experiment ends after a specific number of oscillations. Determine the number of oscillations and read off the time that was needed for these oscillations.

The following results for pendulum (1):

$T_1 = 1.996\,\mathrm{s}$

or $\omega_1 = \frac{2\pi}{T_1} = 3.15\,1/\mathrm{s}$

Task 2 (2/2)

PHYWE

The following results for pendulum (2):

$T_2 = 1.996$

or $\omega_1 = \frac{2\pi}{T_1} = 3.151$/s

This means that the periods of oscillation of the two pendula are identical. If this is not the case, the pendulum lengths must be corrected for the subsequent experiments until the periods of oscillation are identical.

Based on equation (3) and the known pendulum mass m = 1 kg of, the following results for the moment of inertia of the pendulum:

$I_{1/2} = 0.978\,\mathrm{kgm}^2$

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Task 3 (3/8)

PHYWE

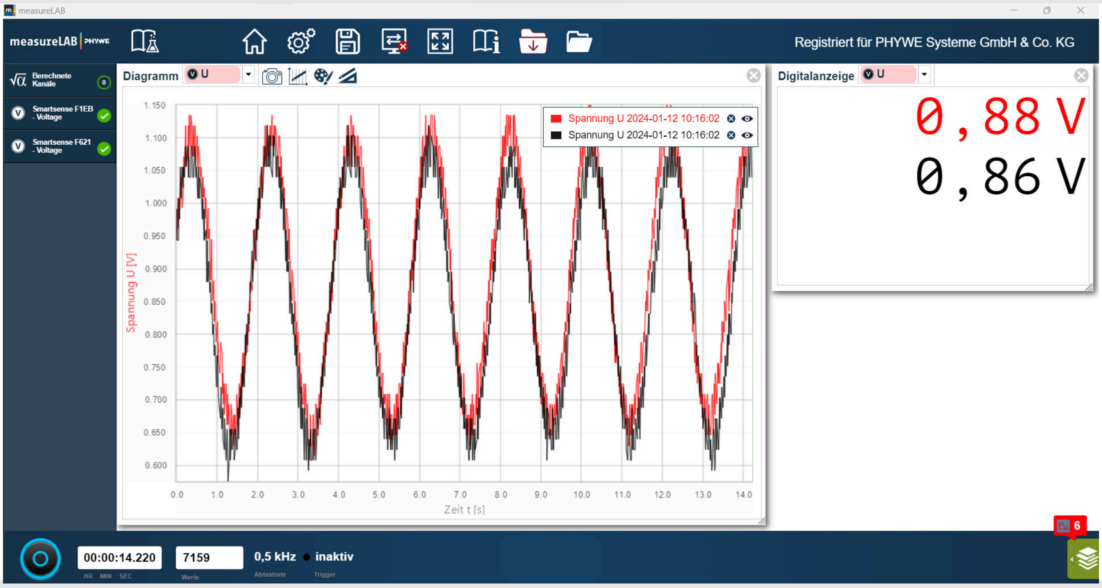

Figure 4: Representation of the deflection of both pendula for the “in phase” oscillation as a function of time t.

Task 3 (4/8)

PHYWE

Figure 5 shows that both pendula oscillate with the same amplitude and frequency ω_g. The y-axis is offset, since the oscillation is not symmetrical with regard to the zero position without a coupling spring. The frequency ω_g is:

$$\omega_g = \frac{2\pi}{T} = \frac{2\pi}{1.993s} = 3.15\,1/s$$

This value corresponds to the value of the characteristic frequency of the uncoupled pendulum. As a result, the theory has been confirmed.

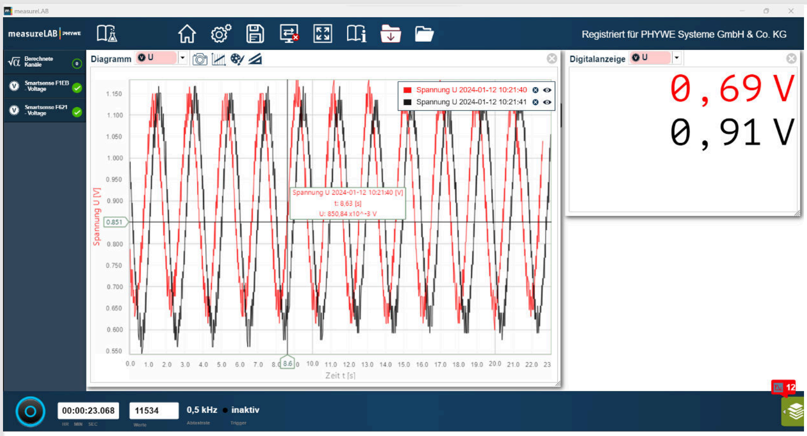

Figure 5: Detail of the deflection of both pendula for the “in phase” oscillation as a function of time t.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Task 3 (5/8)

PHYWE

(B) "antiphase" oscillation

For the "antiphase" oscillation, the process is the same as for A). This leads to:

If we look at a smaller detail, the following results:

Figure 7 shows that both pendula oscillate with nearly similar amplitude and frequency.

ω_c, but with a phase difference of π.

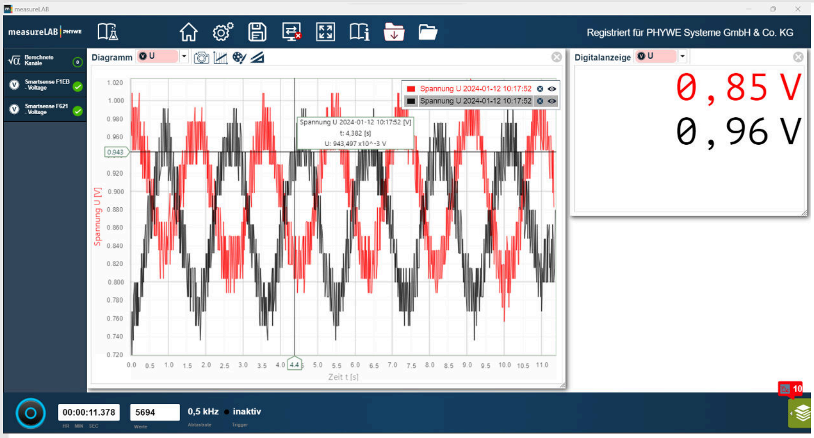

Figure 6: Representation of the deflection of both pendula for the "antiphase" oscillation as a function of time t.

Task 3 (6/8)

PHYWE

The frequency is:

$$\omega_c = \frac{2\pi}{T} = \frac{2\pi}{1.789s} = 3.51\,1/s$$

Based on equation (13), the theoretical frequency is:

$$\omega_c = 3.47\,1/s$$

As a result, the theory has been confirmed.

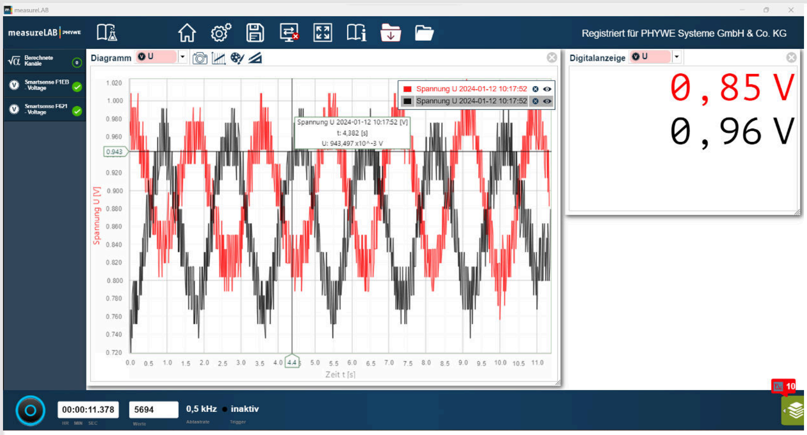

Figure 7: Detail of the deflection of both pendula for the "antiphase" oscillation as a function of time t.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Task 3 (7/8)

PHYWE

(C) beat case

The process for the "beat" case is the same as for A) and B). This leads to:

If we look at a smaller detail, the following results:

Figure 9 shows that there are two oscillations: the to and from oscillation of the pendulum with the frequency P_1 and the reciprocal transfer of the amplitude with the frequency ω_2. They are:

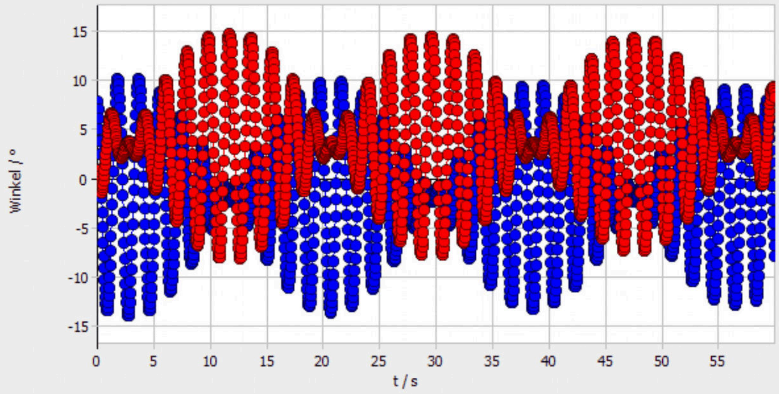

Figure 8: Representation of the deflection of both pendula for the "beat" case as a function of time t.

Task 3 (8/8)

PHYWE

$$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{35.87s} = 0.175 \text{ 1/s}$$

$$\omega_2 = \frac{2\pi}{T_2} = \frac{2\pi}{1.789s} = 3.51 \text{ 1/s}$$

Based on the equations (8a) and (8b) or (14) and (15), respectively, the theoretical frequencies are:

$$\omega_1 = 0.169 \text{ 1/s}$$

$$\omega_2 = 3.32 \text{ 1/s}$$

The experimental frequency values are nearly identical with the theoretical, calculated values.

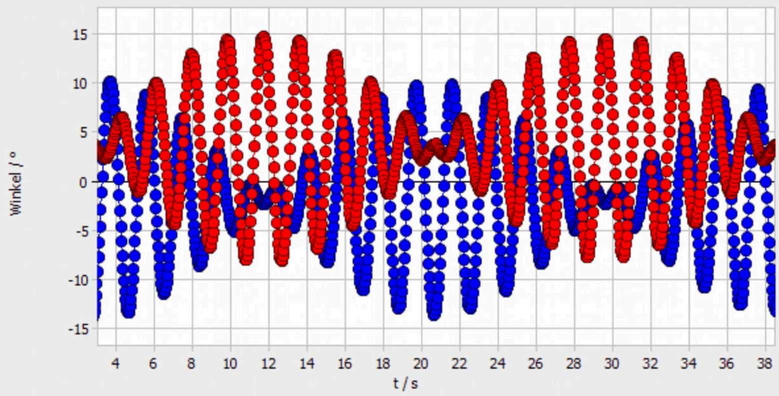

Figure 9: Detail of the deflection of both pendula for the "beat" case as a function of time t.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de