Dispersion and resolving power of a grating spectroscope

Physics | Light & Optics | Diffraction & interference

Difficulty level	Group size	Preparation time	Execution time
-	-	-	-

This content can also be found online at:

http://localhost:1337/c/64d0d25d6d8205000242a36e

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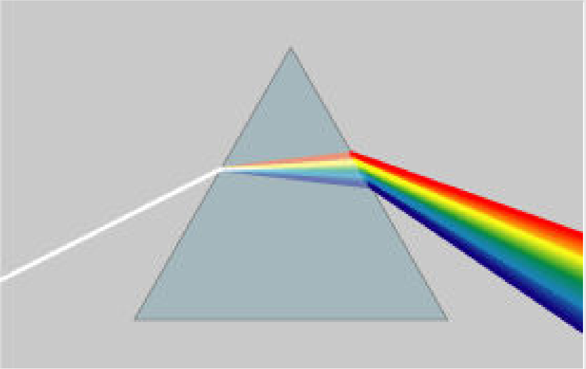
General information

Application



Dispersion of light through a prism

Spectrometers are optical instruments used, to measure properties of light based on the technique selected and the intensity of electromagnetic light being emitted over a range of energies. In principal, a prism or a grating is used in the spectrometer to spread the light from a distant object into a spectrum.

In the field of physics, chemistry and astronomy, spectroscopic analysis is applied to provide information on the composition and physical structure of a matter. For example, astronomers can identify the elements in the stars, as well as the density and the temperature of those elements from spectral lines.

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Other information (1/3)

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Prior knowledge

At the interface between a medium '1' with refractive index n_1 and a medium '2' with refractive index n_2, Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

is applied. The angle of refraction depend on the incident angle and the refractive index of the medium.

Scientific principle

1. The refractive index and the angle of deflection of a light wave depend on its wavelength, $n = n(\lambda)$, as it passes through an optical medium.
2. For gratings, the angles of diffracted orders depend on the grating constant.

Other information (2/3)

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Learning objective

The refractive indicies of liquids, crown glass and flint glass are determined as a function of the wavelength by refraction of light through the prism at minimum deviation. The resolving power of the glass prisms is determined from the dispersion curve. A diffration grating is used to determine the angle of diffraction of the spectral lines of the mercury vapour lamp. The resolving power necessary to separate certain spectral lines is determined.

Tasks

1. Adjusting the spectrometer-goniometer.
2. Determing the refractive index of various liquids in a hollow prism.
3. Determing the refractive index of various glass prism.

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Other information (3/3)

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Tasks

4. Determing the wavelengths of the mercury spectral lines.

5. Demonstrating the relationship between refractive index and wavelength (dispersion curve).

6. Calculating the resolving power of the glass prisms from the slope of the dispersion curves.

7. Determing the grating constant of a Rowland gratin based on the diffraction angle (up to the third oder) of the high intensity spectral lines of mercury.

8. Determing the angular dispersion of a grating.

9. Determing the resolving power required to separate the different Hg-Lines. Comparing with theory.

Safety instructions

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- The common rules of safe experimentation in scientific education are applied in this experiment.

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Theory (1/10)

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Dispersion and resolving power of the prism

The refractive index *n* of a medium is linked to the relative permittivity ϵ_r by the Maxwell relationship

$$n = \sqrt{\epsilon_r \mu_r} \qquad (1)$$

For most substances the permeability is $\mu_r = 1$.

According to Clausius and Mossotti, the following relationship exists between the relative permittivity and the molecular polarizability α of a medium:

$$\alpha = \frac{3\epsilon_0}{N} \cdot \frac{\epsilon - 1}{\epsilon + 2} \qquad (2)$$

where *N* is the concentration of the polarizable molecules and ϵ_0 is the electric field constant.

Theory (2/10)

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The polarizability depends on the frequency $\omega = 2\pi v$ of the incident light. The following is approximately true, beyond the natural frequency $\omega_0 = 2\pi v_0$ of an atom or molecule:

$$\alpha = \frac{e^2}{m} \cdot \frac{1}{\omega^2 - \omega 2} \qquad (3)$$

where *e* is the elementary charge and *m* is the mass of an electron.
When (1) and (3) are substituted in (2),

$$\frac{n^2 - 1}{n^2 + 2} = \frac{e^2 N}{3\epsilon^0 m} \cdot \frac{1}{\omega_0^2 - \omega 2} \qquad (4)$$

is obtained. Although equation (4) only takes one natural frequency into account, this formula adequately describes the decrease in the refractive index as the wavelength increases, outside the range of natural frequencies.

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Theory (3/10)

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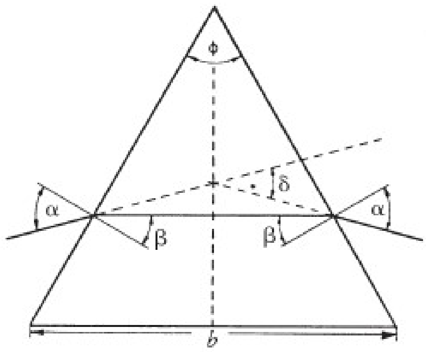
If a ray of light passes symmetrically through a prism, minimum deviation δ occurs. If α is the angle of incidence, β the angle of reflection, and θ the angle of the prism then

$$\sin\alpha = n\sin\beta$$

where $\beta = \frac{\theta}{2}$ and $\delta = 2\alpha - \theta$

From these we obtain

$$n = \frac{sin\frac{\theta+\delta}{2}}{sin\frac{\theta}{2}}$$



Refraction by the prism when the path of a ray is symmetrical.

Theory (4/10)

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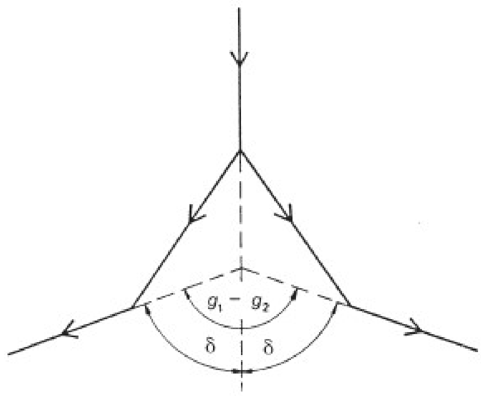
The angle of minimum deviation δ is obtained from the difference between the angles ϕ_1 and ϕ_2 measured at the two different prism position:

$$\delta = \frac{\phi_1 - \phi_2}{2}$$



Refraction by the prism when the path of a ray is symmetrical.

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Theory (5/10)

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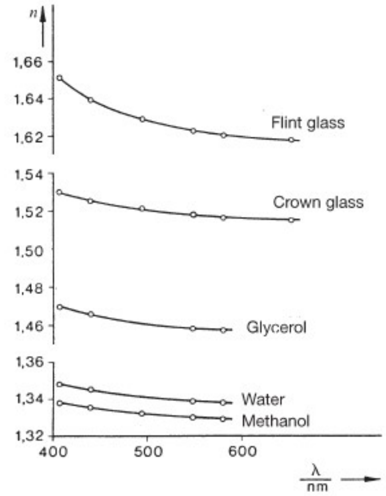
The dispersion curve is determined from the angles measured for the various mercury spectral lines.



Refraction by the prism when the path of a ray is symmetrical.

Theory (6/10)

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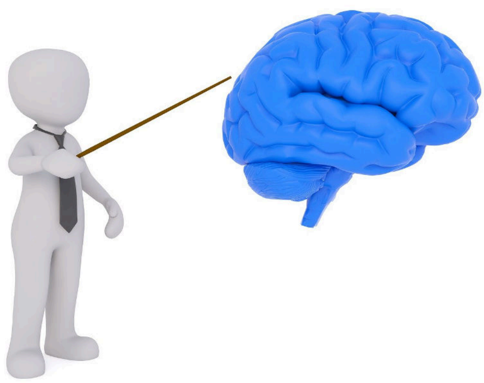
The performance of a spectrometer is characterised by its 'resolving power'. Two wavelengths λ and $\lambda + d\lambda$ are still perceived as separate spectral lines when the principal maximum of line $\lambda + d\lambda$ coincides with the minimum of line λ. The resolving power R is generally defined by the expression:

$$R = \frac{\lambda}{d\lambda}$$

For a prism, the following applies:

$$R = b \cdot \left|\frac{dn}{d\lambda}\right|$$

where b is the base of the prism.



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Theory (7/10)

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Dispersion and resolving power of a grating

The wavelength of the spectral lines are determined with a diffraction grating which is placed in the path of the rays, instead of the prism. For a wavelength λ, the grating constant *G* and angle ϕ at which the first order diffraction pattern appears, the following applies:

$$\lambda = g \; sin \; \phi$$

If monochromatic light with the wavelength λ impinges on a diffraction grating, the intensity diffracted according to the angle ϕ is given by:

$$I(\phi) = I(0) I(\phi) = I(0) = (\frac{\sin u}{u})^2 (\frac{\sin Nv}{v})^2;$$

with $v = \pi \frac{g}{\lambda}$ and $u = \pi \frac{s}{\lambda} sin\phi$, where *s* = width of the slit; *g* = distance between two slits = grating constant; *N*= number of slits)

Theory (8/10)

PHYWE

$$AB - CD = g(\sin\beta - \sin\alpha) = z\lambda$$

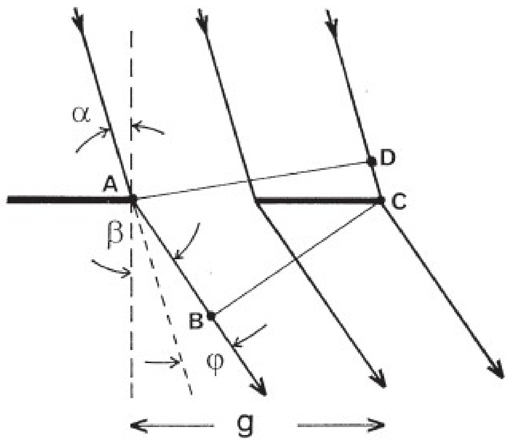
The diffraction direction ϕ of maximum *z* for a given grating fulfils the following relation:

$$v_z = \pi \frac{g}{\lambda} sin \; \phi_z = z\pi \text{ or } sin \; \phi_z = z \frac{\lambda}{g}$$

where *z* = order of diffraction = $\pm$ (0,1,2,...). There are (*N*-1) secondary minima between every two peaks. If light impinges at an angle α against the perpendicular to the grating, the following is valid:

$$\sin\phi + \sin\beta = \frac{z\lambda}{g} = 2 \sin\frac{\phi}{2} \cos\frac{\alpha - \beta}{2}$$

with $\beta - \alpha = \phi$



Diffraction at the grating

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Theory (9/10)

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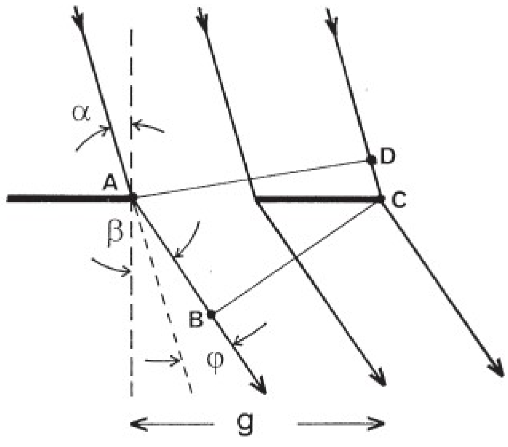
The angle β is considered to be positive when the diffracted and incident beams are on the same side of the perpendicular to the grating. If each angle is on another side of the perpendicular, then β is negative. In case of perpendicular incidence $\alpha = 0$, the following applies:

$$\sin\phi = z\lambda/g$$

The angle of dispersion $\frac{d\phi}{d\lambda}$ of the grating is defined by:

$$\frac{d\phi}{d\lambda} = \frac{z}{g\cos\beta} = \frac{z}{g\cos\alpha}$$

with $\beta = \alpha$, if $\alpha = 0$. Angular dispersion is independent of the angle of incidence and compared to prisms, it remains nearly constant for small diffraction angles.



Diffraction at the grating

Theory (10/10)

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Two spectral lines λ_1 and λ_2 only can be separated if they are so far apart from each other that the peak of λ_1 coincides with the minimum of λ_2 (Rayleigh criterion). The quotient of the average wavelength and the difference between the wavelengths of lines which merely appear separated is called the spectral resolving power.

$$A = \frac{1/2(\lambda_1+\lambda_2)}{(\lambda_2-\lambda_1)} = \frac{\bar{\lambda}}{\triangle\lambda}$$

The following is valid for the spectral resolving power of a diffraction grating:

$$A = zN$$

z = order of diffraction; *N* = effective (illuminated) number of slits.

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Equipment

Position	Material	Item No.	Quantity
1	PHYWE Spectrometer 2	35635-04	1
2	Power supply with holder for spectral lamps E27	08124-97	1
3	Spectral lamp Hg, E27 base	08122-14	1
4	Barrel base expert	02004-00	1

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Setup and procedure

Setup (1/3)

Experimental set-up with glass prism and grid

- Insert the mercury (Hg 100) lamp into the lamp holder. Connect to the power supply and turn on the lamp. The lamp should warm up for about 5 minutes to reach its maximum luminosity. Ensure that the cooling vents on the lamp housing are unimpeded to allow proper air circulation.
- The telescope should be focused to infinite distance. Then the telescope and collimator are adjusted horizontally with the adjusting screws so that the directions of their axes coincide.

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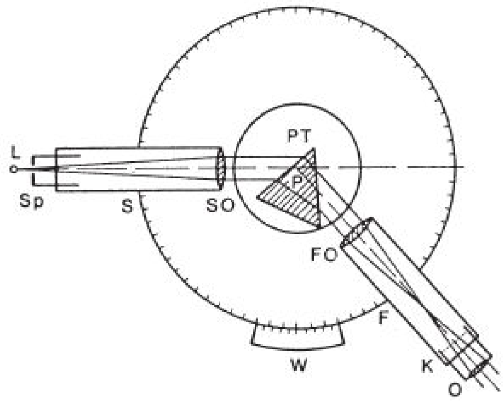
Setup (2/3)

- The Hg-lamp is placed directly before the slit in the drawtube. Adjust the collimator to make the slit image appear sharp while viewing through the telescope (do not refocus telescope).
- Align the telescope cross wires within the slit image. The slit should be adjusted as narrow as possible for increased resolution.



Set-up and path of rays in the spectrometer

Setup (3/3)

- The telescope can be tightened with the lock screw below the lens, and the adjustment screw to the right side of the telescope can be used to more finely move the telescope.
- Darken the room for better observation and use magnifying glass with additional flashflight for accurate scale reading

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Procedure (1/3)

Experiment with glass prism

- Place the prism on the circular table so that the light falls on the one of the prism edges, causing a parallel beam of light to pass through the prism.
- The prism is set to the minimum deviation and the angular position 1 of the telescope is read off on the vernier for each spectral line.
- The prism is then turned so that the light falls on the adjacent surface and is deviated to the opposite side. The angle 2 is now read off for each spectral line, at minimum deviation.

Procedure (2/3)

Experiment with grating

- The diffraction grating should be placed in the the grating holder and fixed onto the circular table. The grating should be set perpendicular to the collimator axis. Align the grating using the three screws on the circular table.
- By moving only the telescope, the diffraction angles of the 6 high intensity Hg spectral lines should be measured.

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Procedure (3/3)

Experiment with grating

- The angle of a spectral line of the same order of diffraction should be measured to both the right and left of the undiffracted light ("zero" order). Make sure to record the telescope position on the outer scale at the "zero" order to use as a reference when measuring the angles to the left and right.
- The first and second order of diffraction lines should be clearly determined, while third order lines should be recognizable to some extent. Align the crosshairs of the telescope to the center of each spectral line, and record the two vernier readings for every angle.

Evaluation

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Evaluation (1/5)

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Spectral region: yellow	$\frac{dn}{d\lambda}/cm^{-1}$	$R = \frac{\lambda}{d\lambda}$
Flint glass	691	2073
Crown glass	377	1131

Spectral region: blue	$\frac{dn}{d\lambda}/cm^{-1}$	$R = \frac{\lambda}{d\lambda}$
Flint glass	2365	7095
Crown glass	1126	3378

The dispersions and resolving powers of glass prisms determined from the dispersion curve

Resolving power of the prism

Resolving power R is determined in the 'yellow' and the 'blue' regions of the spectrum from the slope of the dispersion curve with the prism fully illuminated. Base of the prism must be measured.

Evaluation (2/5)

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Color	λ/nm	$\varphi(1)$	$\varphi(2)$	$\varphi(1)$	$\varphi(2)$	$\overline{\varphi}$	$g/\mu m$
		z = +1		**z = -1**			
Violet	404.7	235^o	73^o	225^o	45^o	14.12^o	1.6586
Blue	435.4	255^o	75^o	224^o	44^o	15.24^o	1.6562

Typical measurement values (diffraction of the first order) to determine the grating constant of the Rowland grating.

Resolving power of the grating

- The diffraction angle φ of a spectral line is calculated from the half angular difference of the corresponding diffraction order $\pm z$.
- The average value $\overline{\varphi}$ is calculated from two values $\varphi(1)$and $\varphi(2)$ which are determined for every angle due to the two verniers.

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Evaluation (3/5)

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Resolving power of the grating

The angular dispersion $\frac{d\varphi}{d\lambda}$ values are determined from the values $d\phi$ and $d\lambda$ which are obtained from the difference between the corresponding values of neighbouring lines.

λ/nm	φ / ^{o}	$d\lambda$ / nm	$d\varphi$ / rad	$\frac{d\varphi}{d\lambda}$ / $10^5 m^{-1}$	$(g\cos\varphi)^{-1}$ / $10^5 m^{-1}$
404.7	14.12				
435.4	15.24	30.75	0.01955	6.36	6.22
491.6	17.25	56.20	0.03490	6.21	6.28

Evaluation of the angular dispersion

Evaluation (4/5)

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Fill in the blank regarding the dispersion of light.

Dispersion is the dependence of ______ of a wave on its ______, as it passes through an optical medium. For example, white light is refracted into its constituent colours as it transmits through a prism, because the ______ of the medium varies with wavelength. Hence, the ______ also varies with wavelength which causes ______.

- angular dispersion
- phase velocity
- wavelength
- angle of refraction
- refractive index

Check

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Evaluation (5/5)

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Which statements are true?

- [ ] The angular dispersion increases with decreasing grating constant
- [ ] Refraction index decrease with increasing wavelength for visible light
- [ ] Red light is diffracted the most through a grating
- [ ] Red light is refracted the most through a prism

Check

Slide	Score / Total
Slide 29: Dispersion of white light	**0/5**
Slide 30: Question regarding optical dispersion	**0/3**

Total Score 0 / 8

Show solutions   Retry

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