Inductance of solenoids with Cobra SMARTsense

Physics | Electricity & Magnetism | Electromagnetism & Induction

Difficulty level	Group size	Preparation time	Execution time
hard	-	45+ minutes	45+ minutes

This content can also be found online at:

http://localhost:1337/c/6494377d2ab9b00002aa4ae0

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General information

Application

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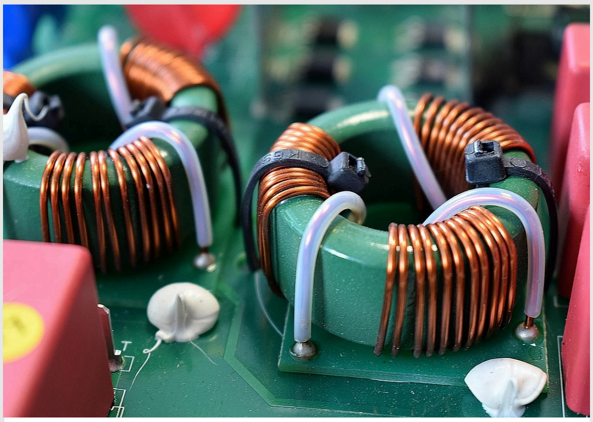

Electromagnetic coils on a board

Inductance is a general property of electrical components, which finds special application in coils. Inductance originates from the connection between the electric and magnetic fields. The self-inductance of an electric circuit relates the rate of change of the electric current $I(t)$ to the electric voltage $U(t)$ over time:

$$U(t) = L \cdot \frac{\mathrm{d}I(t)}{\mathrm{d}t}$$

This relationship between voltage and current can be understood directly with Ampère's law and the law of induction: An electric current generates a magnetic field and the temporal change of the magnetic field induces an electric voltage in the same circuit counteracting this change. Both effects are proportional to the number of turns N.

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Other information (1/2)

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Prior knowledge

Basic knowledge of physical quantities such as current, voltage and resistance should be available. Ideally, the process of electromagnetical induction should be known.

Scientific principle

A square wave voltage of low frequency is applied to an oscillating circuit comprising coil and capacitor of known capacitance. The sudden change of voltage at the both edges of the square wave signal induces a magnetic field in the primary coil, which then couples into the solenoid and triggers a free damped oscillation in the secondary circuit. For different solenoids the natural frequencies of the circuits are measured and therewith the solenoids' inductances are calculated.

Other information (2/2)

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Learning objective

After the successful completion of this experiment you will be able to theoretically understand the phenomenon of the inductance of solenoids. You will also be able to determine the inductance by coupling the solenoid to a capacitor of known capacitance.

Tasks

The natural frequency of the induced oscillation has to be measured for each induction coil. From the natural frequency and the known capacitance, calculate the inductances of the coils and determine the relationships between:

1. Inductance and number of turns
2. Inductance and solenoid length
3. Inductance and solenoid radius

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Safety instructions

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The general instructions for safe experimentation in science lessons apply to this experiment.

Theory (1/4)

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If a current of strength I flows through a cylindrical coil (solenoid) of length l, cross sectional area $A = \pi \cdot r^2$ and number of turns N, a magnetic field is created in the coil. When $I \gg r$ the magnetic field is uniform and the field strength H is given by

$$H = I \cdot \frac{N}{l} \qquad (1)$$

The magnetic flux Φ is given by

$$\Phi = \mu_0 \cdot \mu \cdot H \cdot A \qquad (2)$$

where μ_0 is the magnetic field constant and μ the absolute permeability of the surrounding medium.

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Theory (2/4)

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When this flux changes it induces a voltage between the ends of the coil

$$U_{ind} = -N \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t} = -N \cdot \mu_0 \cdot \mu \cdot A \cdot \frac{\mathrm{d}H}{\mathrm{d}t} = -L \cdot \frac{\mathrm{d}I}{\mathrm{d}t} \qquad (3)$$

$$\text{with} \quad L = \mu_0 \cdot \mu \cdot \pi \cdot \frac{N^2 \cdot r^2}{l} \qquad (4)$$

being the coefficient of inductance of the coil. In practice the inductance of coils with $l > r$ (more specifically $0 < \frac{r}{l} < 1$) can be calculated with great accuracy by the following approximation formula:

$$L = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot \left(\frac{r}{l}\right)^{3/4} \qquad (5)$$

Theory (3/4)

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In the experiment the inductance of various coils is calculated from the period of an oscillating circuit:

$$\omega_0 = \frac{2\pi}{T_{exp}} = \frac{1}{\sqrt{L \cdot C_{tot}}}$$

$C_{tot} = C + C_i$ is the sum of the known capacitor and the input capacitance $C_i \approx 40pF$ of the Cobra 4 XpertLink device, which also induces a damping effect on the oscillatory circuit and causes a negligible shift $(\approx 1\,\%)$ in the resonance frequency. The inductance is therefore represented by

$$L = \frac{T_{exp}^2}{4\pi^2 \cdot C_{tot}} \qquad (6)$$

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Theory (4/4)

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Coil data:

Coil No.	N	$2r\ [mm]$	$l\ [mm]$	Item No.
1	300	40	160	11007-01
2	300	32	160	11007-02
3	300	26	160	11007-03
4	200	40	105	11007-04
5	100	40	53	11007-05
6	150	26	160	11007-06
7	75	26	160	11007-07

Exemplary measured oscillation periods and corresponding calculated inductances (eq. 6) for LC circuit. For comparison, also theoretically predicted inductances are given (eq. 5).

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Equipment

Position	Material	Item No.	Quantity
1	Digital storage oscilloscope with USB, 100 MHz / 2CH, 1GS/s	EAK-P-1404	1
2	PHYWE Digital Function Generator, USB	13654-99	1
3	Induction coils, 1 set (7 coils)	11007-88	1
4	Coil, 1200 turns	06515-01	1
5	Capacitor 470nF/250V, G1	39105-20	1
6	Connection box	06000-00	1
7	Connecting cord, 2 mm-plug, 5A, 500 mm, red	07356-01	2
8	Connecting cord, 2 mm-plug, 5A, 500 mm, blue	07356-04	2
9	Measuring cable BNC to 4 mm banana plug, length 1 m	EAK-MKS-1	2

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Setup and procedure

Setup (1/2)

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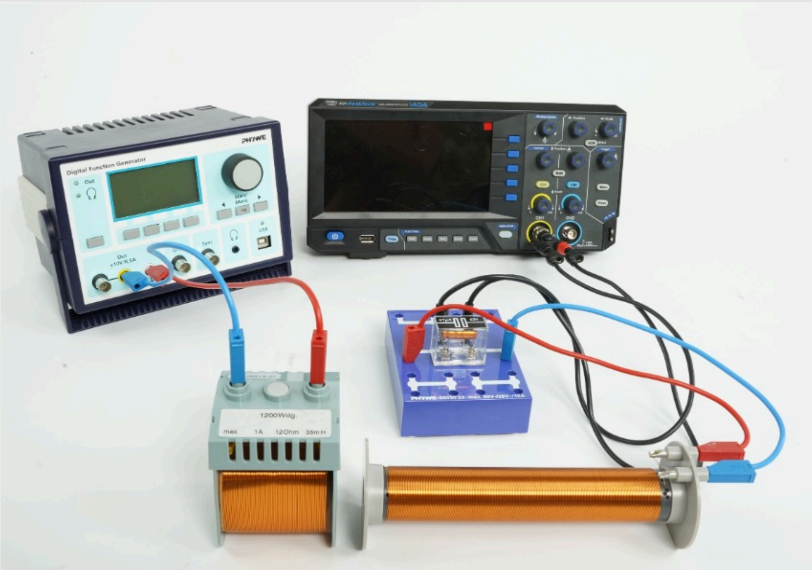

- Build the electrical circuit according to figure: Connect the digital function generator to the primary coil and connect the secondary coil in series to the capacitor (LC circuit), with the oscilloscope connected in parallel.

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Setup (2/2)

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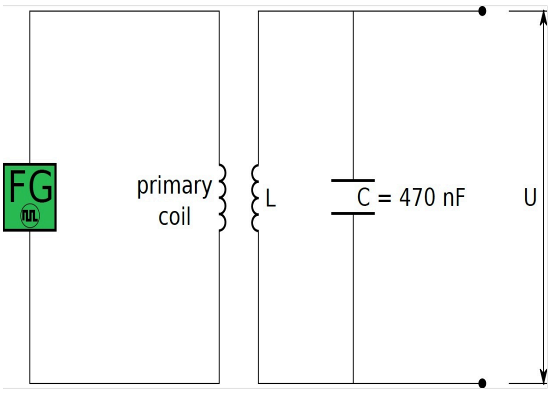

Set-up for inductance measurement

- The digital function generator should have an amplitude of $20\,V$, frequency of $500\,Hz$ and the square wave as signal output.
- The solenoid of the LC circuit has to be aligned carefully with the primary coil so that the magnetic field can couple efficiently from the primary coil into the solenoid. The distance between the two coils should be maximized so that the effect of the excitation coil on the resonant frequency can be disregarded. There should be no iron components in the vicinity of the coils.

Procedure (1/2)

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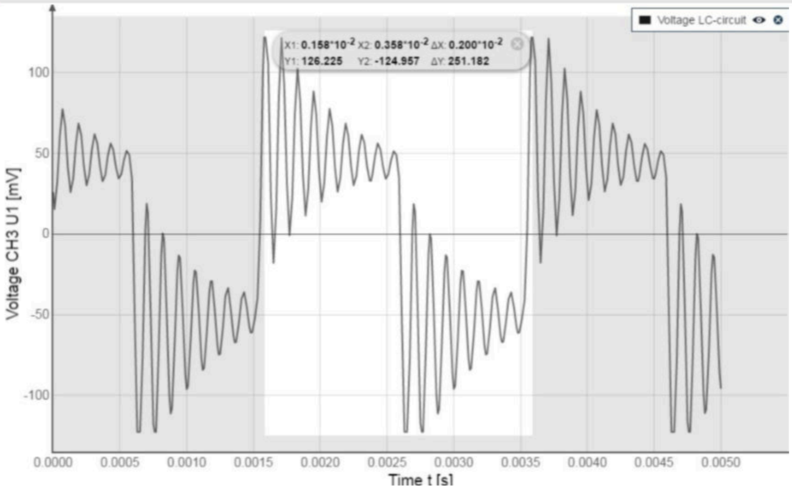

The time for two damped oscillations matches the period time of the square wave signal ($T = 0.002s$).

- Connect the first solenoid to the LC circuit and place it next to the primary coil. Then start a measurement.
- Set the oscilloscope on "Auto-Scale"
- The time for two damped oscillations should match the period time of the square wave signal.
- Zoom in on one damped oscillation and measure the oscillation period T_{exp} of the circuit. Use the measure function and note your result in the evaluation section.

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Procedure (2/2)

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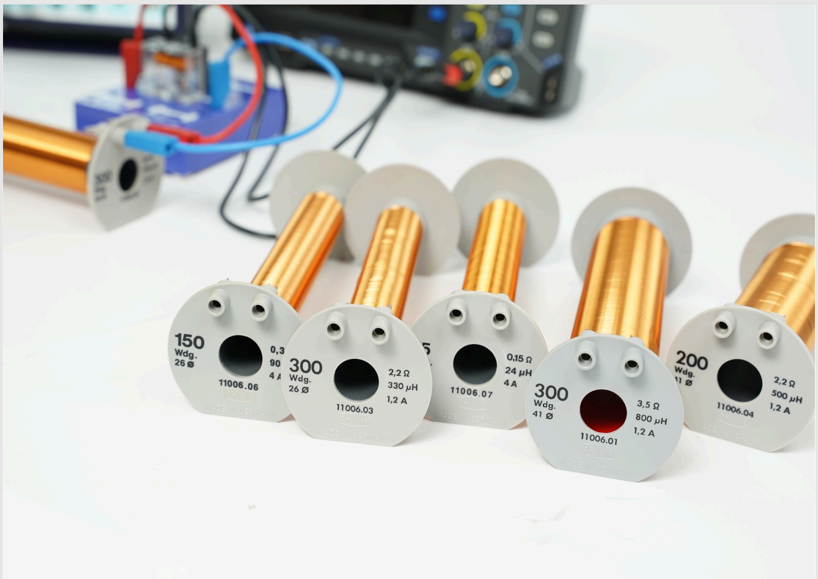

Note: In order to reduce measurement errors you can measure the time over several periods of the oscillation and divide your result by the number of periods you have taken.

- Repeat these steps for all the different solenoids. Note all resulting measurement values in the table in the evaluation section.

Evaluation (1/9)

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Coil No.	T_{exp} $[\mu s]$	L_{exp} $[\mu H]$	L_{theo} $[\mu H]$
1			
2			
3			
4			
5			
6			
7			

- Note the measured oscillation period T_{exp} of the circuit for the different coils in the table and calculate the experimental inductance L_{exp}.
- Calculate the theoretical inductance L_{theo} according to the parameters of the coils.
- Compare your results to the exemplary data provided to cross-check for major measurement mistakes:

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Evaluation (2/9)

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Note: To analyse the resulting data, you will need to fit it to different mathematical functions. You can choose any software you like to do this step a well-performing software is gnuplot. The following coils provide the relationships between inductance and 1. number of turns N , 2. length l and 3. radius r that we are investigating:

1. Coils no. 3, 6, 7

2. Coils no. 1, 4, 5

3. Coils no. 1, 2, 3

The relationship between inductance and number of turns found in task 1 must also be used to solve task 2.

Evaluation (3/9)

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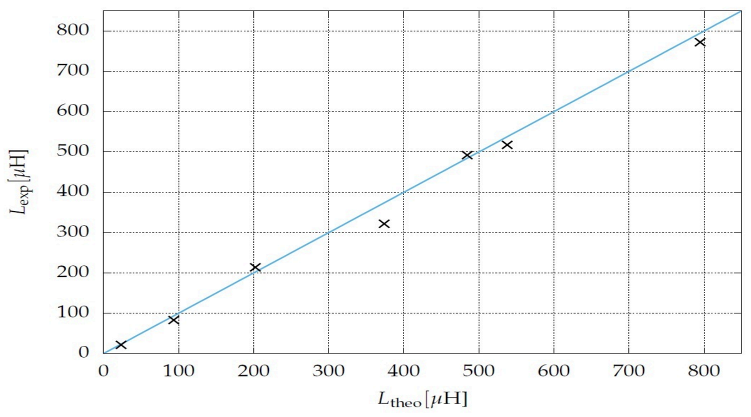

Comparison of experimentally obtained values for the inductances of the LC circuit solenoids with theoretically calculated values in a scatter diagram.

Plot the resulting experimental values against the theoretical ones. Therefore generate two data sets, that contain the data. After generating the data sets, select them both in the data pool and choose the option "diagram" to display a diagram.

In the diagram choose the data set for the theoretical values as x-axis. Select 'Points' to display a scatter plot of your data. Go back and generate a regression line. The slope should match very well with 1.

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Evaluation (4/9)

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1. Task: Determine the coils' relationship between inductance and number of turns.

- To determine the relationship between inductance and number of turns consider coils with identical radius and length but different number of turns. Here, coils no. 3, 6 and 7 meet these requirements.

As done before for the inductances, generate the respective data sets (Inductance L for coils no. 3,6 and 7 and the corresponding number of turns N). Subsequently, select both data sets to be displayed in a diagram, choose the number of turns as x-axis and select the representation of the data as points. Lastly perform a curve fitting according to the instructions on the following page.

Evaluation (5/9)

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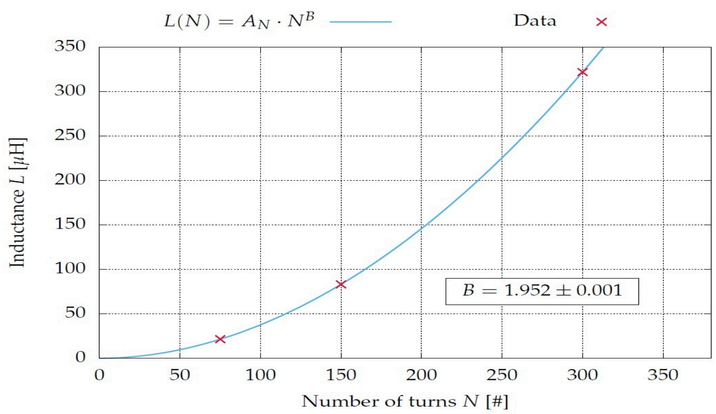

Relation between inductance and number of turns.

Fit the resulting data to the expression $L_{exp} = a \cdot x^b$.

The exemplary data yields

$b = 1.952 \pm 0.001$

which is in excellent agreement with the theoretical value $b_{theo} = 2$ (eq. 5).

In the figure the corresponding inductances are plotted in dependence of the number of turns for the exemplary data.

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Evaluation (6/9)

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2. Task: Determine the coils' relationship between inductance and length of coil.

- To determine the relationship between inductance and length of coil, only consider coils with identical radius but different lengths. The coils no. 1, 4 and 5 meet these requirements. As the relation between inductance and number of turns is already known, the inductances can be normalized by the number of turns. Therefore consider the relationship between inductance normalized by turn number squared and the length of coil.

Evaluation (7/9)

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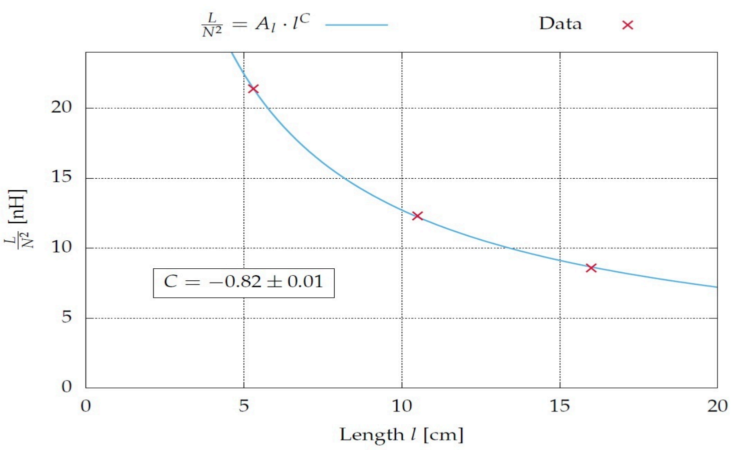

Relation between inductance and length of coil

Fit the resulting data to the expression $L_{exp}/N^2 = a_l \cdot x^c$.

The exemplary data yields

$c = -0.82 \pm 0.01$

which is in fair agreement with the theoretical value $c_{theo} = -0.75$ (eq. 5).

In the figure the corresponding normalizes inductances are plotted in dependence of the coil length for the exemplary data.

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Evaluation (8/9)

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3. Task: Determine the coils' relationships between inductance and radius of the coils.

- To determine the relationship between inductance and radius of the coils, only consider coils with identical lengths but different radii. The coils no. 1, 2 and 3 meet these requirements.

Evaluation (9/9)

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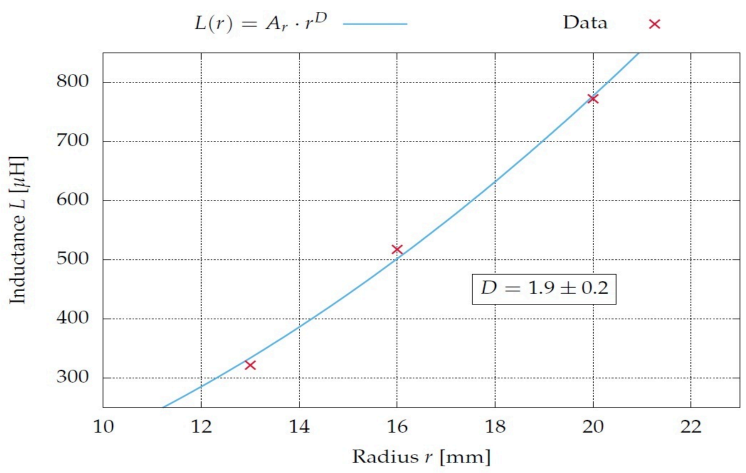

Relation between inductance and radius of the coil.

Fit the resulting data to the expression $L_{exp} = a_r \cdot x^d$.

The exemplary data yields

$d = 1.9 \pm 0.2$

which is in good agreement with the theoretical value $d_{theo} = 1.75$ (eq. 5).

In the figure the corresponding inductances are plotted in dependence of the coil radius for the exemplary data.

Show solutions

Retry

Export text

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