Characteristic X-ray lines of different anode materials / Moseley's law

Physics	Modern Physics	Production & use of X-rays

Difficulty level	Group size	Preparation time	Execution time
hard	2	15 minutes	90 minutes

This content can also be found online at:

https://www.curriculab.de/c/5f60869a7e9d5b0003e1e759

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General information

Application

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Setup

Most applications of X rays are based on their ability to pass through matter. Since this ability is dependent on the density of the matter, imaging of the interior of objects and even peaple becomes possible. This has wide usage in fields such as medicine or security.

Other information (1/2)   **PHYWE**

Prior knowledge

The prior knowledge required for this experiment is found in the Theory section.

Main principle

Moseley's law describes the relationship between the energy of the K_α lines of characteristic X-ray spectra and the atomic number. In this experiment, the characteristic X-ray lines of various different anode materials are determined in order to verify Moseley's law.

Other information (2/2)   **PHYWE**

Learning objective

The goal of this experiment is to get to investigate the monochromatic characteristic X-radiation of copper.

Tasks

1. Record the X-ray spectra of the three X-ray tubes.

2. Determine the wavelengths and frequencies of the characteristic X-ray lines based on the Bragg angles of the lines.

3. Create the Moseley lines and determine the Rydberg constant and screening constant.

Theory (1/4)

H. G. J. Moseley discovered the relationship between the energy of the K_α lines of characteristic X-ray spectra and the atomic number. If the root of the frequency of the K_β line is plotted as a function of the atomic number Z of the anode material, a straight line results.

Based on this straight line, the order of the elements in the periodic table of elements was specified in a definite manner for the very first time. The element hafnium (Hf) (Z = 72) that had been unknown hitherto, was represented as a gap on Moseley's straight line. Following the discovery of hafnium and the recording of the X-ray spectrum, the element fitted right into this gap, which substantiated Moseley's findings.

Theory (2/4)

The following is valid for the binding energy E_n of an electron on a shell with the principal quantum number n:

$$E_n = -\frac{m_e e^4}{8\epsilon_0^2 h^2}(Z - \sigma)^2 \quad \text{where (n = 1, 2, 3, ...)} \quad (1)$$

Electron mass : $\quad m_e = 9.1091 \cdot 10^{-31} \text{ kg}$

Elementary charge: $\quad e = 1.6021 \cdot 10^{-19} \text{ As}$

Plank's constant: $\quad h = 6.6256 \cdot 10^{-34} \text{ Js}$

Dielectric constant: $\quad \epsilon_0 = 8.8544 \cdot 10^{-12} \text{ F/m}$

Atomic number: $\quad$ Z

Screening constant: $\quad \sigma$

Theory (3/4)   PHYWE

During the transition of an electron from L shell to a free space on the K shell of an atom, the energy that is released can be converted into X-radiation. The frequency f of this quantum can be determined with the aid of equation (1):

$$f = \frac{\Delta E}{h} = \frac{m_e e^4}{8\epsilon_0^2 h^2}(Z-\sigma)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ Moseley's law} \quad (2)$$

$$= f_R \cdot (Z-\sigma)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\left(f_R = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 3.2899 \cdot 10^{15}\, s-1 = \text{Rydbergfrequency}\right)$$

With $n_1 = 1$ and $n_2 = 2$, it follows from (2) that:

$$\sqrt{f} = \tfrac{1}{2}\sqrt{3 f_R}\,(Z-\sigma)$$

Theory (4/4)   PHYWE

If the interplanar spacing d of the analyser crystal is known, the glancing angles θ of the characteristic K_α and K_β lines can be used to determine the wavelengths λ of the lines based on Bragg's law.

$$2d \sin(\theta) = n\lambda \quad (4) \quad (\text{d: interplanar spacing; n = 1, 2, 3,...})$$

(LiF(200) interplanar spacing d = 201.4 pm)

The associated frequencies f of the characteristic lines result from:

$$c = \lambda \cdot f \text{ (velocity of light } c = 2.9979 \cdot 10^8 \text{m/s}\text{)}$$
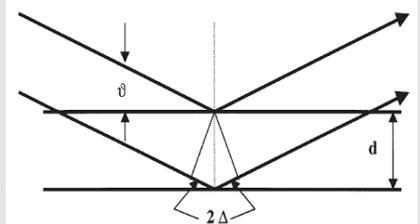

Fig. 1: Bragg scattering on a pair of lattice planes

Equipment

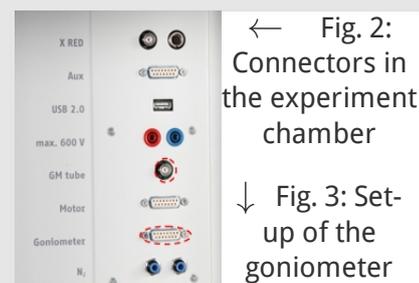| Position	Material	Item No.	Quantity
1	XR 4.0 expert unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR4 X-ray Plug-in Cu tube	09057-51	1
4	XR4 X-ray Plug-in Mo tube	09057-61	1
5	XR4 X-ray Plug-in Fe tube	09057-71	1
6	XRC 4.0 X-ray characteristics upgrade set	09135-88	1

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Setup and Procedure

Setup

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Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig. 2). The goniometer block with the analyser crystal should be located at the end position on the right-hand side. Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube (see Fig. 3). Insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit.

← Fig. 2: Connectors in the experiment chamber

↓ Fig. 3: Set-up of the goniometer

For calibration: Make sure, that the correct crystal is entered in the goniometer parameters. Then, select "Menu", "Goniometer", "Autocalibration". The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks.
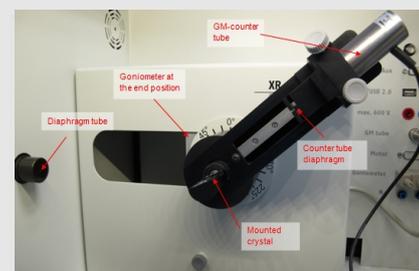
Procedure (1/3)

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- Connect the X-ray unit via the USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Figure 4).

- Start the "measure" program. A virtual X-ray unit will be displayed on the screen.

- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.

Fig. 4: Connection of the computer

Procedure (2/3)

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Fig. 5: Part of the user interface of the software

- Click the experiment chamber (see the red marking in Figure 5) to change the parameters for the experiment. Start angle: 3° - 4°. Record the spectra at least up to the second-order characteristic lines.

- If you click the X-ray tube (see the red marking in Figure 5), you can change the voltage and current of the X-ray tube. Select the parameters as shown in Fig. 6.
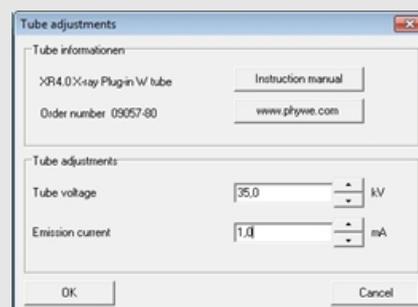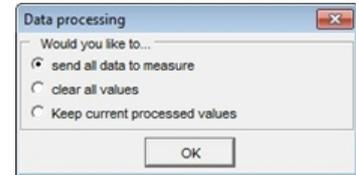

Fig 6: Voltage and current settings

Procedure (3/3)

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- Start the measurement by clicking the red circle:

- After the measurement, the following window appears:

- Select the first item and confirm by clicking OK. The measured values will now be transferred directly to the "measure" software.

- At the end of this manual, you will find a brief introduction to the evaluation of the resulting spectra.

Overview of the goniometer and X-ray unit settings:

- 1:2 coupling mode

- Gate time 2 s; angle step width 0.1°

- Start angle: 3° - 4°. Record the spectra at least up the second-order characteristic lines.

- Anode voltage U_A = 35 kV; anode curren I_A = 1 mA

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Evaluation

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Task 1

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Task 1: Record the X-ray spectra of the three X-ray tubes.

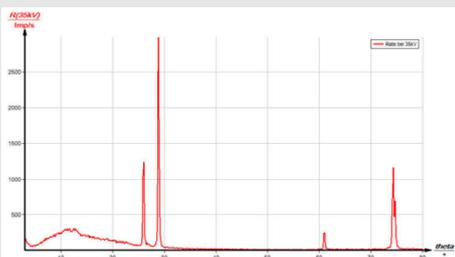The X-ray spectra of iron, copper, and molybdenum with the LiF crystal as the analyser are shown in Figure 7a-7c.

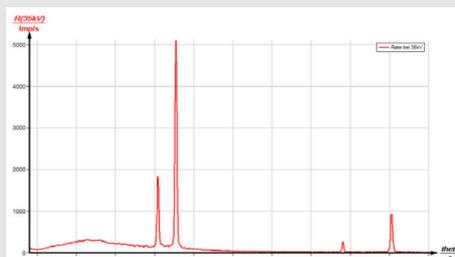
Fig. 7a: X-ray spectrum of iron (Z = 26)
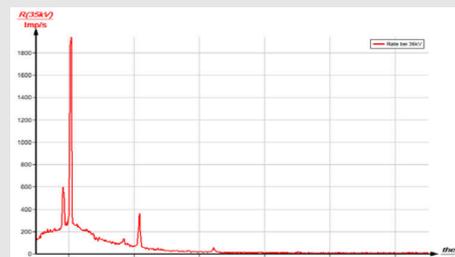

Fig. 7b: X-ray spectrum of copper (Z = 29)

Fig. 7c: X-ray spectrum of molybdenum (Z = 42)

Task 2

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Task 2 Determine the wavelengths and frequencies of the characteristic X-ray lines based on the Bragg angles of the lines.

Table 1 shows the θ values of the characteristic K_α and K_β lines of the three anode materials that were determined based on the spectra as well as the associated wavelength and frequency values that were determined with the aid of equations (3) and (4).

	n = 1		n = 2		n = 3			
	ϑ/°	λ/pm	ϑ/°	λ/pm	ϑ/°	λ/pm	$\overline{\lambda}$ /pm	$\sqrt{f(K_{\alpha/\beta})}/10^{8}$
K_α lines								
Fe (Z = 26)	28.9	194.7	74.3	193.9	-	-	194.3	12.42
Cu (Z = 29)	22.6	154.1	50.2	154.9	-	-	154.5	13.93
Mo (Z = 42)	10.2	70.4	20.8	71.2	32.1	71.3	71.0	20.55
K_β lines								
Fe (Z = 26)	25.8	175.3	60.9	176.0	-	-	175.7	13.06
Cu (Z = 29)	20.4	140.4	43.9	139.6	-	-	140.0	14.63
Mo (Z = 42)	9.2	64.4	18.5	63.9	28.2	63.4	63.9	21.66

Table 1

Task 3

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Task 3: Create the Moseley lines and determine the Rydberg constant and screening constant.

Figure 8 shows the two Moseley lines that result from the calculated values (see table 1). The mean value of the two gradients

$$\mathrm{m} = 0.5 \cdot 10^8 \, \mathrm{s}^{-1/2} = \tfrac{1}{2}\sqrt{3 \mathrm{f_R}}$$

leads to the Rydberg frequency $\mathrm{f_R} = 3.33 \cdot 10^{15} \, \mathrm{s}^{-1}$. The screening constant can be determined with the aid of equation (3): $\sigma_{2,1} \approx 1$.
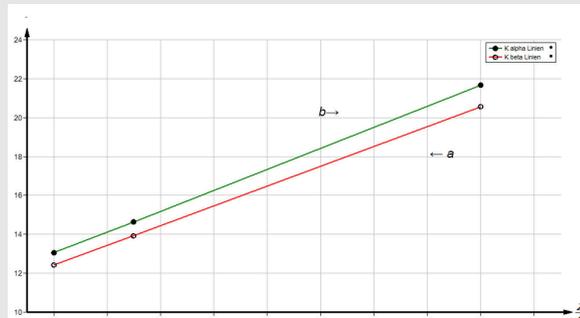

Fig. 8: Moseley lines: Curve a: transition $\mathrm{n_2 \rightarrow n_1}$ (K_α line) Curve b: transition $\mathrm{n_3 \rightarrow n_1}$ (K_β line)

Note

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On the tab "Measurement", click "Enter data manually". Then, enter the number of measurement values into the corresponding field and enter also the number of channels. Click "Next" and enter the values into the corresponding fields (under "Number" the x-values, i.e. the atomic number and the calculated values into the channels). The resulting straight lines nearly superimpose each other since they are scaled individually based on the left axis. In order to change this, click the button ⊿ on the top bar and select "Fit collectively". Right-click the spectrum if you would like to display the data table or change the display options. You can, for example, change the names of the channels or select a certain line type. If you click one of the lines and then "Analysis" –> "Show slope", the slope of the selected straight line can be displayed as shown in Figure 8.
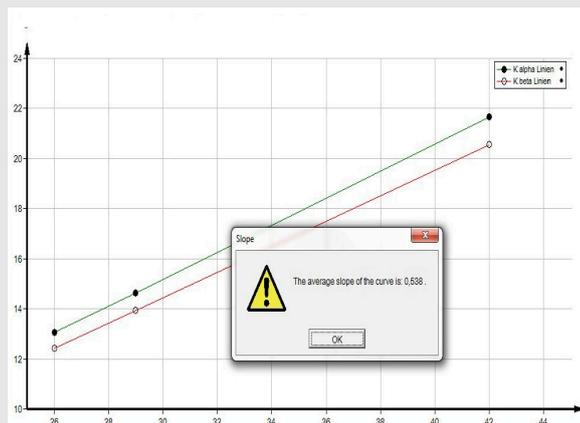

Fig. 9: Evaluation of the measurement values with the "measure" software

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